Hydrodynamic aspects of lava tube caves in the Krongno area, Dak Nong province, The Central Highlands, Vietnam

“Tsutomu Honda1, Hiroshi Tachihara1, Hirofumi Miyashita1 and Akira Miyazaki2
1 NPO Vulcano-Speleological Society, Japan: mer4beau939tha@gmail.com
2 Vietnam National Museum of Nature (VNMN), Vietnam Academy of Science and Technology (VAST)

Abstract
From 2012 to 2015, the joint team of the Vietnam National Museum of Nature (VNMN) belonging to the Vietnam Academy of Science and Technology (VAST) and NPO Vulcano-Speleological Society of Japan carried out a survey for volcanic caves in the lava flowed from the Chu B’Luk volcano located in Krongno district, Dak Nong province, The Central Highlands of Vietnam. As a result of the survey, the joint team explored 18 lava tube caves, among which 11 lava tube caves were measured and mapped. The inner wall (ribbed wall) and ceiling (lava stalactite) observation of lava tube cave, together with the measurement of the height and slope angle for some caves are carried out. By using two hydrodynamic models for these observations, two physical properties for lava yield strength and surface tension were estimated from these results.

From lava tube cave height and slope angle, the flow in the lava tube is modeled by Bingham fluid flowing in the inclined cylindrical pipe with gravity potential. Then, the condition of the cave formation is formulated and this formulation was applied to estimate the yield strength in the studied area. Gravity, lava density, slope angle and cave height are the decisive parameters that determine the Bingham yield strength of lava. For lava tube Cave C2, the estimated yield strength is 2.3x10^4 dyne/cm^2, which shows a reasonable value as yield strength of basaltic lava.

From ceiling and wall surface observation, the role of surface tension of lava on the formation of lava stalactite or ribbed wall is analyzed by a hydrodynamic instability model of lava boundary layer attached to the ceiling or side wall. The surface tension 560–990 dyne/cm estimated from this model for Cave C3 and Cave B14, shows a reasonable value as surface tension of basaltic lava.

1. Introduction

The Chu B’Luk volcano is located in Krongno district, Dak Nong province, The Central Highlands of Vietnam (Fig.1). It is one of the continental volcanoes that blew a large amount of soft lava having a silicic acid weight fraction of 48.3 to 52.4% (1). The Vietnam National Museum of Nature (VNMN), and the NPO Vulcano Speleological Society conducted a joint survey of the lava tube cave in the lava flow ranging from the Chu B’Luk crater to the Dray Sap Waterfall area from 2012 to 2015 (2–9). As a result, 18 lava tube caves were discovered and surveyed, and 11 caves of which were measured and mapped, and the total extended distance was 4832.5 m at the time of January 2015. The location of the lava caves including lava caves recently surveyed (20 caves in sum total up to now at 2018) (10), are also shown in Fig.1. Fig.2 shows a typical measurement result for the lava tube cave for Cave C2. We present findings obtained on the physical properties (yield strength and surface tension) of lava deduced by cave geometry and internal observation.

2. Hydrodynamic models of Bingham fluid flow for lava tubes

A considered schematic for lava tube flow is indicated on Fig.3 where H=2R is tube diameter (or tube height), R is radius of the lava tube, and α is slope angle of the lava tube.
Fig. 1 Distribution map of lava caves in Krongno Volcano Geopark, Dak Nong province, Vietnam

Fig. 2 Horizontal and vertical cross section of lava tube Cave C2
Fig. 3 shows the lava spouted from a crater goes down a slope and forms a lava tube. The flow in the lava tube is controlled by the gravity. After the termination of eruption (drain back of magma), a hollow is formed in the tube producing a “lava tube cave” in which the lava in the tube could be drained out by the gravity (free flow). The hydrodynamic model for lava tubes with the flow speed distribution in the tube is shown in Fig. 4.

![Fig. 3 Schematic of lava tube flow](image)

![Fig. 4 The flow speed distribution in the tube](image)

The equation of the flow speed distribution $u$ in the tube is shown as below:

For $\tau_w = (\rho g \sin \alpha)R/2 \geq f_B$

$$u = (R-r_B)^2(\rho g \sin \alpha)/4\eta_B$$  \hspace{1cm} r < r_B$$

$$u = [R^2-r^2-2r_B(R-r)](\rho g \sin \alpha)/4\eta_B$$  \hspace{1cm} r \geq r_B$$

Here, $\tau_w$ is shear stress on the tube wall surface, $r_B$ is radius where shear stress is equal to $f_B$, $f_B$ is Bingham yield strength, $\eta_B$ is Bingham viscosity, $g$ is the gravity force and $\rho$ is lava density. Critical condition for lava tube cave formation is: for $H=2R$, $H=4f_B/(\rho g \sin \alpha)$, then, $f_B= H(\rho g \sin \alpha)/4$.

### 3. Bingham yield strength estimated from lava tube cave height

Lots of lava tube caves are found between Chu B’Luk volcano and Dray Sap Waterfall in a straight line distance of about 9 km. Among them, the lava tube cave C2 is used as a typical lava tube cave. The cavern height $H$ of the C2 cave (Fig. 2) is about 10 m, the total length is 402.2 m and the elevation difference between upper and lower extremities is 15 m, so the inclination angle is $\alpha=2.1$ degrees.

![Fig. 5 Inside of C2](image)
4. Hydrodynamic instability model and estimate of surface tension from lava stalactite and ribbed wall

Fig.7, 8, 9. show a general feature of the inside of lava tube cave. Lava stalactites are positioned periodically on the surface of the ceiling wall or side wall. From the periodical pitch of the stalactites, we can obtain the surface tension of the lava\(^{(13,14)}\). The pitch will be the critical wave length of the occurrence of instability of thin liquid film attached on the surface of the ceiling of the lava tube cave as shown in Fig. 10. The pitch \(P\) is shown as: \(P=2\pi(\gamma/g\rho)^{1/2}\), where \(\gamma\) is surface tension of liquid, \(\rho\) is density of liquid, \(g\) is gravity acceleration. From the pitch of lava stalactites on the roof surface, the surface tension of lava \(\gamma= P^2 g\rho /4\pi^2\) is determined. If there is a superposition of the lateral and vertical surface flow, the ribbed wall will appear and keep the same pitch as that of lava stalactite. As for the surface tension calculated from the pitch of lava stalactites on the roof surface of Cave C3 or ribbed wall of Cave B14 (\(P=3\) to 4cm) (see Fig. 11 and Fig. 12), the surface tension of lava was determined as 560～990 dyne/cm. The estimated surface tension matches with the experimental results by melting the lava in the Laboratory\(^{(15)}\). It is considered that it is a reasonable value as basaltic lava.
5. Summary:

The following two tables show findings of hydrodynamic aspects obtained on the physical properties (yield strength and surface tension) of lava by cave geometry and internal structure observation.

**Table 1: Yield strength of lava flow of Chu B’luk Volcano**

<table>
<thead>
<tr>
<th>Item relating to yield strength</th>
<th>Numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of lava tube cave C2</td>
<td>402.2 m</td>
</tr>
<tr>
<td>Height of lava tube cave: H</td>
<td>10 m</td>
</tr>
<tr>
<td>Difference in elevation between upper and lower extremities</td>
<td>15 m</td>
</tr>
<tr>
<td>Slope angle $\alpha$ of the cave C2 (Inclination of the Cave C2)</td>
<td>2.1 degree</td>
</tr>
<tr>
<td>Yield strength: $fb=H(\rho g \sin \alpha)/4$, $\rho$:2.5$g/cm^3$, $g$:980cm/sec$^2$</td>
<td>$2.3\times10^4$ dyne/cm$^2$</td>
</tr>
</tbody>
</table>

**Table 2: Surface tension of lava flow of Chu B’Luk Volcano**

<table>
<thead>
<tr>
<th>Item related to surface tension</th>
<th>Numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch of lava stalactite of ceiling: Cave C3 and ribbed wall of Cave B14</td>
<td>3~4cm</td>
</tr>
<tr>
<td>Surface tension: $\gamma=P^2 \frac{g\rho}{4\pi}$, $\rho$:2.5$g/cm^3$, $g$:980cm/sec$^2$</td>
<td>560 to 990 dyne/cm</td>
</tr>
</tbody>
</table>

These physical values of lava flow of Chu B’Luk volcano (elevation: 601m) are reasonable value as basaltic lava. This lava of high fluidity is similar to that of Kilaeua volcano and Piton de la Fournaise. The drilling survey results shows that lava flow thickness is 120~140m. This suggest that the eruption of lava flow from Chu B’Luk volcano was like a flood.

**References:**


(5) La The Phuc, Hiroshi Tachihara, Tsutomu Honda, Truong Quang Quy, Luong Thi Tuat(2015): The geological heritage of unique volcanic cave in Dak Nong has been discovered and established the record. Journal of Geology, Series A, 349, 1 - 2/2015, 28-38
(8) Tsutomu Honda, Hiroshi Tachihara, La The Phuc, Luong Thi Tuat, Truong Quang Quy(2016): Vietnam Volcanic Cave Survey Project Report,
(14) T. Honda(2016): The role of surface tension on the formation of lava stalactite and lava stalagmite 17th Internat. Vulcanospeleology Symposium, Hawaii